

CP violation in sbottom decays

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Abstract

We study CP asymmetries in two-body decays of bottom squarks into charginos and tops. These asymmetries probe the SUSY CP phases of the sbottom and the chargino sector in the Minimal Supersymmetric Standard Model. We identify the MSSM parameter space where the CP asymmetries are sizeable, and analyze the feasibility of their observation at the LHC. As a result, potentially detectable CP asymmetries in sbottom decays are found, which motivates further detailed experimental studies for probing the SUSY CP phases.

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1 Introduction

Supersymmetry (SUSY) [1] is a well motivated theory to extend the Standard Model (SM) of particle physics. SUSY models are not only favored by gauge coupling unification and naturalness considerations, but are also attractive from the cosmological point of view. For instance the lightest SUSY particle (LSP) is a good dark matter candidate if it is stable, massive and weakly interacting [2, 3]. SUSY models can also provide new sources of CP violation [4]. In the Minimal Supersymmetric Standard Model (MSSM) [1], the complex parameters are conventionally chosen to be the Higgsino mass parameter μ , the U(1) and SU(3) gaugino mass parameters M_1 and M_3 , respectively, and the trilinear scalar coupling parameters A_f of the third generation sfermions ($f = b, t, \tau$),

$$\mu = |\mu|e^{i\phi_\mu}, \quad M_1 = |M_1|e^{i\phi_1}, \quad M_3 = |M_3|e^{i\phi_3}, \quad A_f = |A_f|e^{i\phi_{A_f}}. \quad (1)$$

These phases contribute to the electric dipole moments (EDMs), in particular to those of Thallium [5], Mercury [6], the neutron [7], and the deuteron [8], which can be beyond their current experimental upper bounds [5–9]. The experimental limits generally restrict the CP phases to be smaller than $\pi/10$, in particular the phase ϕ_μ [10, 11]. However, the extent to which the EDMs can constrain the SUSY phases strongly depends on the considered model and its parameters [10–16].

As shown for example in Ref [16], due to cancellations among different contributions to the EDMs, only isolated points in the CP phase space can give large CP-violating signals at the LHC. It is important to search for these signals, since the cancellations could be a consequence of a deeper model that correlates the phases. In addition, the existing EDM bounds can also be fulfilled by including lepton flavor violating couplings in the slepton sector [14]. This is important when considering for example SUSY Seesaw models, where CP violation in the slepton sector is connected to the neutrino sector and Leptogenesis [17, 18].

Thus measurements of SUSY CP observables outside the low energy EDM sector are necessary to independently determine or constrain the phases. In particular, the phases of the trilinear scalar coupling parameters A_f have a significant impact on the MSSM Higgs sector [19]. Loop effects, dominantly mediated by third generation squarks, can generate large CP-violating scalar-pseudoscalar transitions among the neutral Higgs bosons [20, 21]. As a result, the lightest Higgs boson with a mass of order 10 GeV or 45 GeV [22] cannot be excluded by measurements at LEP [23]. The fundamental properties and the phenomenology of CP-violating neutral Higgs boson mixings have been investigated in detail in the literature [24, 25].

The phases can also drastically change other SUSY particle masses, their cross sections, branching ratios [26–29], and longitudinal polarizations of final fermions [30]. Although such CP-even observables can be very sensitive to the CP phases (the observables can change by an order of magnitude and more), CP-odd (T-odd) observables have to be measured for a direct evidence of CP violation. CP-odd observables are, for example, rate asymmetries of cross sections, distributions, and partial decay widths [31]. However, these observables require the presence of absorptive phases,

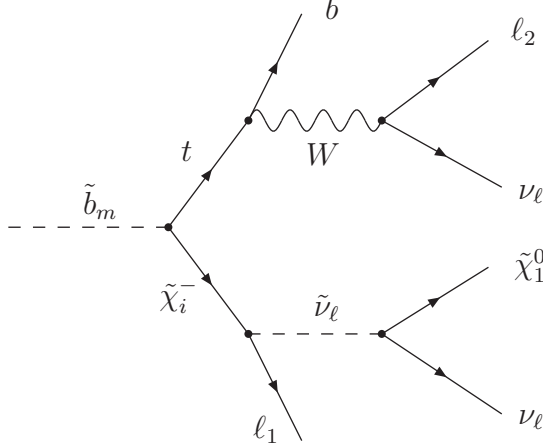


Figure 1: Schematic picture of bottom squark decay.

e.g. from loops. Thus they usually do not exceed the size of 10%, unless they are resonantly enhanced [21, 25, 32, 33].

Larger CP asymmetries in particle decay chains can be obtained with triple products of final particle momenta [34]. They already appear at tree level due to spin correlations. Triple product asymmetries have been intensively studied in the production and decay of neutralinos [35–39] and charginos [39–42] at the ILC [43], also using transversely polarized beams [44]. At the LHC [45], triple product asymmetries have been studied for the decays of neutralinos [37, 46, 47], stops [16, 48, 49], and sbottoms [50]. For recent reviews, see Ref. [51].

In this paper, we thus study CP asymmetries in two-body decays of a sbottom,

$$\tilde{b}_m \rightarrow t + \tilde{\chi}_i^-; \quad m = 1, 2; \quad i = 1, 2; \quad (2)$$

followed by the subsequent two-body decay of the chargino,

$$\tilde{\chi}_i^- \rightarrow \ell_1^- + \tilde{\nu}_\ell^*, \quad \ell = e, \mu, \quad (3)$$

with the invisible sneutrino decay $\tilde{\nu}_\ell^* \rightarrow \tilde{\chi}_1^0 \bar{\nu}_\ell$, and the subsequent top decay

$$t \rightarrow b + W^+; \quad W^+ \rightarrow \nu_\ell + \ell_2^+; \quad \ell = e, \mu, \quad (4)$$

see Figure 1 for a schematic picture of the entire sbottom decay chain. The CP-sensitive spin-spin correlations of the sbottom decay allow us to probe the phase of the coupling parameter A_b , and the phase of the higgsino mass parameter μ . The phases of the trilinear scalar coupling parameters of the third generation sfermions are rather unconstrained by the EDMs [10, 15]. Therefore it is appealing to study CP asymmetries in squark decays at high energy colliders like the LHC [45] or ILC [43]. The third generation sfermions also have a rich phenomenology due to a sizeable mixing of left and right states.

CP asymmetries based on triple products have already been studied in two-body decays of sbottoms [50], however in their rest frame only. At colliders like the LHC, the particles are highly boosted, which will generally reduce the triple product asymmetries [16, 46, 47, 49]. We thus will also include the sbottom production at the LHC.

In addition, we will also study asymmetries which base on epsilon products. Those asymmetries are boost invariant, and thus provide the largest possible asymmetries. We calculate the amplitude squared for the entire sbottom decay in the spin-density matrix formalism [52]. The compact form of the amplitude squared allows us to identify the optimal CP observables for sbottom decays, an asymmetry, that has not been studied in Ref. [50].

In Section 2, we identify the CP-sensitive parts in the amplitude squared, define the CP asymmetries in bottom squark decays, and discuss their dependence on the complex sbottom-top-chargino couplings. We analyze their MSSM parameter dependence in a SUSY benchmark scenario. In Section 3, we discuss sbottom production and their boost distribution at the LHC. We give lower bounds on the required LHC luminosities to observe the asymmetries over their statistical fluctuations. We summarize and give our conclusions in Section 4. In the Appendix, we review sbottom mixing with complex parameters, give the phase space, and calculate the sbottom decay amplitudes in the spin-density matrix formalism.

2 CP asymmetries in bottom squark decays

In this Section, we identify the CP-sensitive parts in the amplitude squared of the entire two-body decay chain of the bottom squark, see Eqs. (2)-(4) and Figure 1. In order to probe these parts, we define CP asymmetries of epsilon and triple products of the particle momenta. Explicit expressions for the squared amplitude, Lagrangians, couplings, and phase-space elements are summarized in the Appendix.

2.1 T-odd products

The amplitude squared $|T|^2$ for the sbottom decay chain, see Figure 1, can be decomposed into contributions from the top spin correlations, the chargino spin correlations, the top-chargino spin-spin correlations, and an unpolarized part, see Eq. (E.50). Since we have a two-body decay of a scalar particle, a CP-sensitive part can only originate from the spin-spin correlations. It is given by the last summand in Eq. (E.55), which is proportional to

$$|T|^2 \supset \text{Im}\{l_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}[p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}], \quad m, i = 1, 2. \quad (5)$$

The left and right couplings $k_{mi}^{\tilde{b}}$ and $l_{mi}^{\tilde{b}}$, respectively, are defined through the \tilde{b}_m - t - $\tilde{\chi}_i^\pm$ Lagrangian [48],

$$\mathcal{L}_{t\tilde{b}\tilde{\chi}^\pm} = g \bar{t} (l_{mi}^{\tilde{b}} P_R + k_{mi}^{\tilde{b}} P_L) \tilde{\chi}_i^\pm \tilde{b}_m + \text{h.c.}, \quad (6)$$

see Appendix C. These couplings depend on the mixing in the sbottom and chargino sector, and thus on the CP phases ϕ_{A_b} and ϕ_μ . The imaginary part of the coupling product, $\text{Im}\{l_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}$, in Eq. (5) is multiplied by a T-odd epsilon product \mathcal{E} , for which we use the short hand notation

$$\mathcal{E} \equiv [p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}] \equiv \varepsilon_{\mu\nu\alpha\beta} p_{\tilde{b}}^\mu p_t^\nu p_{\ell_1}^\alpha p_{\ell_2}^\beta, \quad (7)$$

with the convention $\varepsilon_{0123} = 1$. Since each of the spatial components of the four-momenta changes sign under a naive time transformation, $t \rightarrow -t$, this product is T-odd. Due to CPT invariance, T-odd products are related to CP-odd observables.

2.2 T-odd asymmetries

The task is to define an observable, that projects out the CP-sensitive part of the spin-spin correlation term from the amplitude squared. This can be achieved by defining for the T-odd product \mathcal{E} , Eq. (7), the T-odd asymmetry of the partial sbottom decay width Γ [16],

$$\mathcal{A} = \frac{\Gamma(\mathcal{E} > 0) - \Gamma(\mathcal{E} < 0)}{\Gamma(\mathcal{E} > 0) + \Gamma(\mathcal{E} < 0)} = \frac{\int \text{Sign}[\mathcal{E}] |T|^2 d\text{Lips}}{\int |T|^2 d\text{Lips}}, \quad (8)$$

with the amplitude squared $|T|^2$, and the Lorentz invariant phase-space element $d\text{Lips}$, such that $\int |T|^2 d\text{Lips} / (2m_{\tilde{b}}) = \Gamma$. The T-odd asymmetry is also CP-odd, if absorptive phases (from higher order final-state interactions or finite-width effects) can be neglected [34].

In general, largest asymmetries are obtained by using the epsilon product \mathcal{E} , see Eqs. (5) and (7), that matches the kinematic dependence of the CP-sensitive terms in the amplitude squared. In the literature, this technique is sometimes referred to *optimal observables* [53]. Other possible combinations³ of momenta, $\mathcal{E} = [p_{\tilde{b}}, p_b, p_{\ell_1}, p_{\ell_2}]$, or $\mathcal{E} = [p_{\tilde{b}}, p_t, p_b, p_{\ell_1}]$, lead to smaller asymmetries, see Ref. [50], and Fig. 6.

Triple products of three spatial momenta can also be used to define asymmetries [34, 50]. In the sbottom rest frame, $p_{\tilde{b}}^\mu = (m_{\tilde{b}}, \mathbf{0})$, the epsilon product is

$$[p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}] = m_{\tilde{b}} \mathbf{p}_t \cdot (\mathbf{p}_{\ell_1} \times \mathbf{p}_{\ell_2}) \equiv m_{\tilde{b}} \mathcal{T}. \quad (9)$$

That triple product will give the largest asymmetries in the sbottom rest frame, and the other combinations of momenta for \mathcal{T} lead to smaller asymmetries.

Note that the asymmetries of an epsilon product \mathcal{E} are by construction Lorentz invariant whereas those constructed with a triple product \mathcal{T} are not [16, 46]. The triple product asymmetries will therefore depend on the sbottom boost, $\beta_{\tilde{b}} = |\mathbf{p}_{\tilde{b}}|/E_{\tilde{b}}$, and are generally reduced if not evaluated in the sbottom rest frame. We will discuss the impact of the sbottom boost on the asymmetries at the LHC in Section 3.

³ Note that momenta from both the decay products of the chargino *and* the top have to be included to obtain non-vanishing asymmetries [50]. Otherwise the CP-sensitive top-chargino spin-spin correlations are lost. If for example momenta from the top decay are not taken into account, only CP asymmetries from the chargino decay can be obtained. In addition, a three-body decay is required, (or a two-body decay via an on-shell W boson [40]) to probe the phases ϕ_μ, ϕ_1 of the chargino/neutralino system. The asymmetries are then of the order of 10%, which is typical for chargino three-body decays [42], and also neutralino three-body decays [36, 38, 46, 47, 49].

Table 1: MSSM scenario. The mass dimension parameters are given in GeV.

$ \mu $	M_2	$M_{\tilde{D}}$	$M_{\tilde{Q}}$	$\tan \beta$	A_b	ϕ_{A_b}	$\phi_\mu = \phi_{M_1}$
200	250	400	420	5	1200	$\frac{1}{5}\pi$	0

2.3 Parameter and phase dependence

In order to analyze the phase and MSSM parameter dependence of the asymmetry \mathcal{A} , Eq. (8), we insert the explicit form of the amplitude squared in the spin-density formalism. As shown in Appendix F, we obtain

$$\mathcal{A} = \eta \frac{\int \text{Sign}(\mathcal{E})(p_b \cdot p_{\nu_\ell}) [p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}] d\text{Lips}}{(p_{\chi_i^\pm} \cdot p_{\ell_1}) \int (p_t \cdot p_{\ell_2})(p_b \cdot p_{\nu_\ell}) d\text{Lips}}, \quad (10)$$

with $(p_{\chi_i^\pm} \cdot p_{\ell_1}) = (m_{\chi_i^\pm}^2 - m_{\nu_\ell}^2)/2$, and the coupling function

$$\eta = \frac{\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}}{\frac{1}{2} \left(|\tilde{l}_{mi}^{\tilde{b}}|^2 + |k_{mi}^{\tilde{b}}|^2 \right) \frac{m_{\tilde{b}}^2 - m_{\chi_i^\pm}^2 - m_t^2}{2m_{\chi_i^\pm} m_t} - \text{Re}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}}. \quad (11)$$

The asymmetry can thus be separated into a kinematical part and the effective coupling factor η . That factor is approximately independent of the particle masses, but governs the main dependence on the CP phases, and on the parameters of the sbottom-top-chargino couplings $\tilde{l}_{mi}^{\tilde{b}}$, $k_{mi}^{\tilde{b}}$. To qualitatively understand this dependence, we expand [50]

$$\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\} = c_m Y_t \text{Im}\{U_{i1}^* V_{i2}^*\} - \frac{1}{2} d_m Y_t Y_b \sin(2\theta_{\tilde{b}}) \text{Im}\{U_{i2}^* V_{i2}^* e^{-i\phi_{\tilde{b}}}\}, \quad (12)$$

with the Yukawa couplings Y_t and Y_b , see Eq. (C.14), the short hand notation $c_1 = \cos^2 \theta_{\tilde{b}}$, $c_2 = \sin^2 \theta_{\tilde{b}}$, $d_1 = 1$, $d_2 = -1$, the sbottom mixing angle $\theta_{\tilde{b}}$, Eq. (A.8), and the CP phase $\phi_{\tilde{b}} = \arg[A_b - \mu^* \tan \beta]$, Eq. (A.4), of the sbottom system. The imaginary part of the products of the sbottom-top-chargino couplings $\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}$ is sensitive to the phases ϕ_μ and ϕ_{A_b} , and can be large due to the Yukawa couplings Y_t, Y_b . For $\phi_\mu = 0$ or π , the chargino diagonalization matrices U, V are real, and [50]

$$\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\} \propto \sin(2\theta_{\tilde{b}}) \sin(\phi_{\tilde{b}}), \quad (13)$$

In particular, for $\phi_\mu = 0$ or π , $\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}$ shows a $\sin(\phi_{A_b})$ behaviour, and is maximal at $\sin(\phi_{A_b}) \approx \pi/2, 3\pi/2$. However, the maxima of the asymmetries correspond to the maxima of η , which are shifted away from $\phi_{A_b} = \pi/2, 3\pi/2$. This is due to the influence of the denominator of η , see Eq. (F.65), which has a cosine-like dependence on ϕ_{A_b} , and η will be approximately maximal for $|\tilde{l}_{mi}^{\tilde{b}}| \approx |k_{mi}^{\tilde{b}}|$.

Table 2: SUSY particle masses for the benchmark scenario of Table 1 ($\ell = e, \mu$).

$m_{\tilde{b}_1} = 400 \text{ GeV}$	$m_{\tilde{b}_2} = 424 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} = 158 \text{ GeV}$	$m_{\tilde{\chi}_2^\pm} = 301 \text{ GeV}$
$m_{\tilde{\chi}_1^0} = 108 \text{ GeV}$	$m_{\tilde{\chi}_2^0} = 172 \text{ GeV}$	$m_{\tilde{\chi}_3^0} = 206 \text{ GeV}$	$m_{\tilde{\chi}_4^0} = 302 \text{ GeV}$
$m_{\tilde{\nu}_\ell} = 137 \text{ GeV}$	$m_{\tilde{\tau}_1} = 152 \text{ GeV}$	$m_{\tilde{\ell}_R} = 156 \text{ GeV}$	$m_{\tilde{\ell}_L} = 157 \text{ GeV}$

2.4 Numerical analysis

To quantitatively study the asymmetry \mathcal{A} , Eq. (8), we analyze the behaviour of the coupling factor η in more detail, and define a benchmark scenario in Table 1. We give the relevant resulting SUSY masses in Table 2. We fix the soft-breaking parameters in the slepton sector by $M_{\tilde{E}}^\ell = M_{\tilde{L}}^\ell = 150 \text{ GeV}$ for $\ell = e, \mu, \tau$, to enable the subsequent chargino decay $\tilde{\chi}_1^\pm \rightarrow \ell_1^\pm \tilde{\nu}_\ell^{(*)}$. We take stau mixing into account, and fix the trilinear scalar coupling parameter $A_\tau = 250 \text{ GeV}$. Since its phase does not contribute to the CP asymmetry, we set it to $\phi_{A_\tau} = 0$, as well as $\phi_1 = 0$, which is the phase of the U(1) gaugino mass parameter M_1 . In order to reduce the number of parameters, we further use the GUT inspired⁴ relation $|M_1| = 5/3 M_2 \tan^2 \theta_w$.

We will study the decay of the lighter sbottom into the lightest chargino, $\tilde{b}_1 \rightarrow t \tilde{\chi}_1^\pm$, which gives the dominant contribution to the asymmetries. For our reference scenario as defined in Table 1, the corresponding branching ratios are $\text{BR}(\tilde{b}_1 \rightarrow t \tilde{\chi}_1^\pm) = 19\%$ and $\text{BR}(\tilde{\chi}_1^\pm \rightarrow e^\pm \tilde{\nu}_e) = 33\%$. Other decay channels yield much smaller asymmetries for the scenario as defined in Table 1. Further, we will focus on the largest (Lorentz invariant) asymmetry for the optimal T-odd product $\mathcal{E} = [p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}]$, see Eq. (7). Note that in the sbottom rest frame, this asymmetry is equivalent to the triple product asymmetry with $\mathcal{T} = \mathbf{p}_t \cdot (\mathbf{p}_{\ell_1} \times \mathbf{p}_{\ell_2})$, see Eq. (9). We use the short hand notation $\mathcal{A}(t\ell_1\ell_2)$ for the asymmetry, to indicate the momenta used for the triple product.

In Figure 2, we show the dependence of the coupling factor η , Eq. (11), and its corresponding asymmetry $\mathcal{A}(t\ell_1\ell_2)$, Eq. (10), on the two CP phases ϕ_{A_b} and ϕ_μ , for the SUSY scenario of Table 1. We clearly see that the maxima of $\mathcal{A}(t\ell_1\ell_2) \approx \pm 40\%$ are not necessarily obtained for maximal CP phases, $\phi_{A_b, \mu} = \pi/2, 3\pi/2$. The reason is that the phase dependence of \mathcal{A} is almost governed by the coupling factor η , shown in the left panel of Figure 2. Thus the asymmetry can be sizeable even for small values of the phases, favored by EDM constraints, which in particular constrain ϕ_μ . For example, the asymmetry has a maximum of $\mathcal{A}(t\ell_1\ell_2) \approx \pm 40\%$ at small $\phi_\mu \approx \pm 0.2\pi$, for $\phi_{A_b} = 0$. The positions of the maxima will remain when including the effects of the sbottom boost, as we will discuss in Section 3.1. Figure 2 motivates the choice $\phi_{A_b} = 0.2\pi$, $\phi_\mu = 0$, in our benchmark scenario, Table 1.

⁴Note that this choice also can significantly constrain the neutralino sector [57].

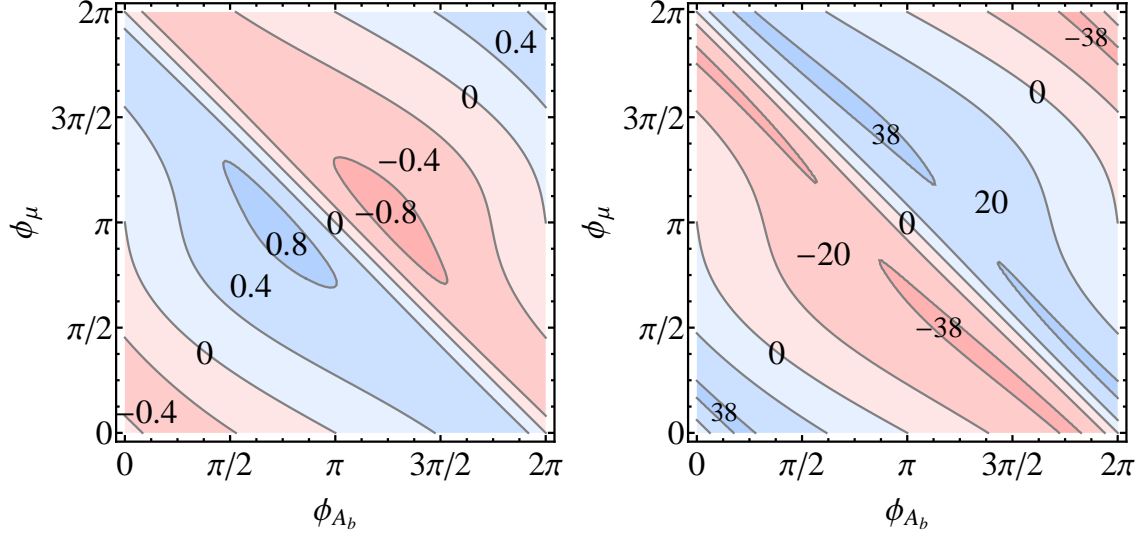


Figure 2: Phase dependence of the CP-odd coupling factor η , Eq. (11), for sbottom decay $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$ (left), and the corresponding CP asymmetry $\mathcal{A}(t\ell_1\ell_2)$, Eq. (8), in percent (right), for the subsequent two-body decay chain $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$, and $t \rightarrow b\nu_\ell \ell_2^+$, see Figure 1, in the \tilde{b}_1 rest frame. The SUSY parameters are given in Table 1.

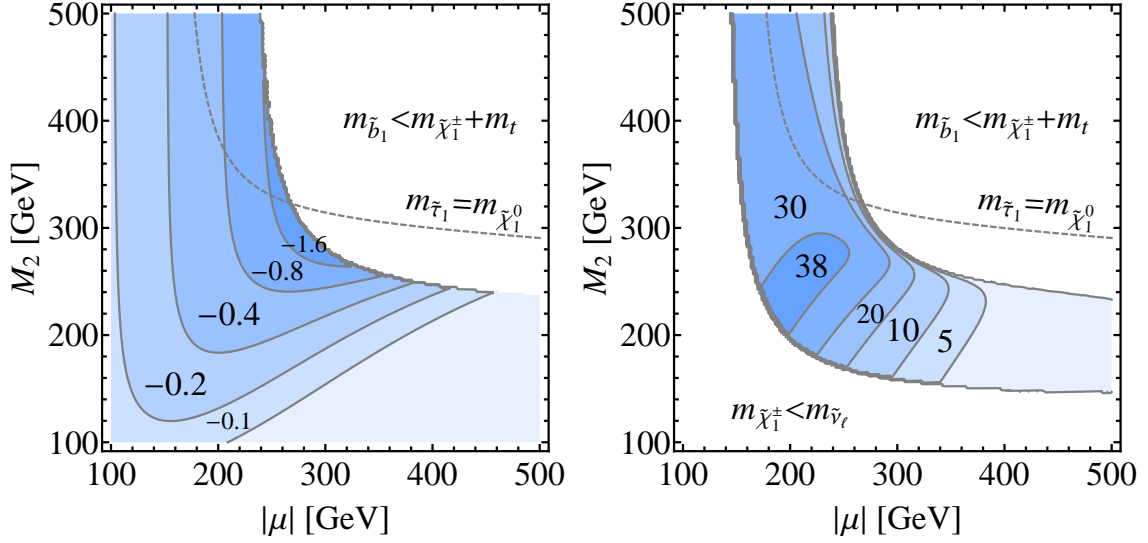


Figure 3: Contour lines in the μ - M_2 plane of the CP-odd coupling factor η , Eq. (11), for sbottom decay $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$ (left), and the corresponding CP asymmetry $\mathcal{A}(t\ell_1\ell_2)$, Eq. (8), in percent (right), for the subsequent two-body decay chain $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$, and $t \rightarrow b\nu_\ell \ell_2^+$, see Figure 1, in the sbottom rest frame. The SUSY parameters are given in Table 1. The area above the contour lines is kinematically forbidden by $m_{\tilde{b}_1} < m_{\tilde{\chi}_1^+} + m_t$, the area below the contour lines of the asymmetry (right) is forbidden by $m_{\tilde{\chi}_1^\pm} < m_{\tilde{\nu}_\ell}$. The area above the dashed line is excluded by $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$.

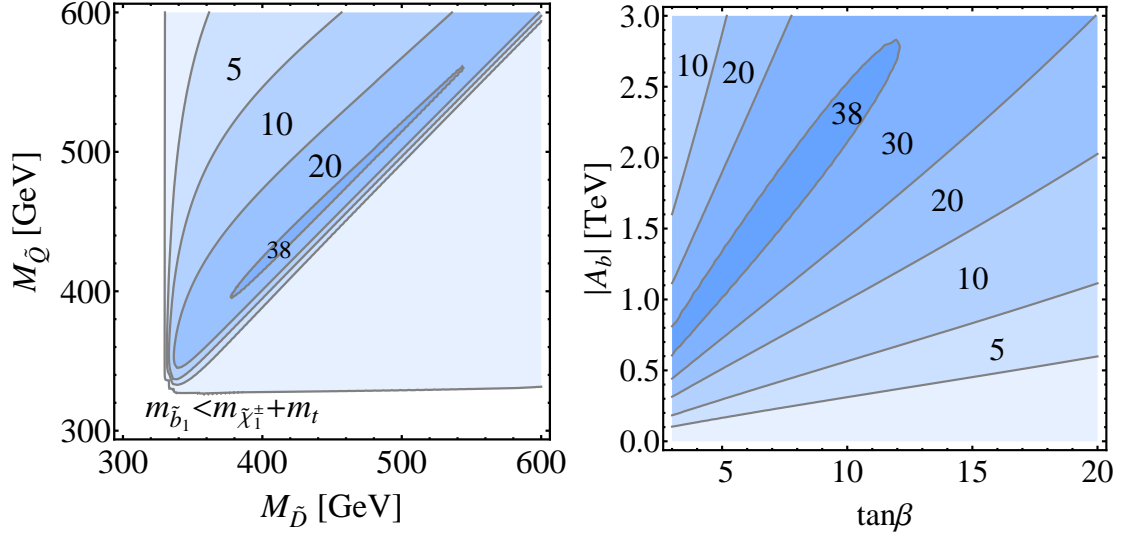


Figure 4: Contour lines of the CP asymmetry $\mathcal{A}(t\ell_1\ell_2)$, Eq. (8), in percent for sbottom decay $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$, followed by the subsequent two-body decay chain $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$, and $t \rightarrow b\nu_\ell \ell_2^+$, in the plane of the soft breaking parameters $M_{\tilde{Q}}$, $M_{\tilde{D}}$ (left), and in the $\tan\beta$ - $|A_b|$ plane (right). The SUSY parameters are given in Table 1.

In Figure 3, we present the dependence of the asymmetry (left) and the coupling factor η (right) on the chargino mixing, which is mainly determined by the higgsino and gaugino parameters μ and M_2 , respectively. We find large values of the coupling factor η and the asymmetry for mixed and higgsino-like charginos. The kinematical boundaries for the sbottom decay $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$, and the chargino decay $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$ are indicated in Figure 3.

The size of the asymmetry \mathcal{A} , Eq. (10), strongly depends on the sbottom mixing, see the mixing matrix Eq. (A.1), in Appendix A. The mixing is determined by the soft-breaking parameters $M_{\tilde{Q}}$, $M_{\tilde{D}}$, on the diagonal, and $m_b(A_b - \mu^* \tan\beta)$ on the off-diagonal entries of the sbottom mixing matrix. In Figure 4 (left), we see that the asymmetry is maximal in the region of the level crossing $M_{\tilde{Q}} \approx M_{\tilde{D}}$, where also $\text{Im}\{l_{11}^{\tilde{b}}(k_{11}^{\tilde{b}})^*\}$ is maximal, see Eq. (12). However, although the imaginary part quickly drops for increasing $M_{\tilde{Q}} > M_{\tilde{D}}$, the asymmetry is still sizeable in that region, since the couplings still fulfill $|l_{11}^{\tilde{b}}| \approx |k_{11}^{\tilde{b}}|$.

In Figure 4 (right), we show in the $\tan\beta$ - $|A_b|$ plane contour lines of the asymmetry, which peaks at $|A_b| \approx |\mu| \tan\beta$, where $|l_{11}^{\tilde{b}}| \approx |k_{11}^{\tilde{b}}|$. Whereas the imaginary part of the couplings, $\text{Im}\{l_{11}^{\tilde{b}}(k_{11}^{\tilde{b}})^*\}$, steadily grows with increasing $|A_b| > |\mu| \tan\beta$, see Eq. (12), the asymmetry is reduced in that region. This is since the sbottom width $\Gamma(\tilde{b}_1 \rightarrow t\tilde{\chi}_1^\pm)$ increases, which enters in the denominator of the asymmetry, see Eq. (8). Thus from Figure 4, we can observe that the contribution from the CP-even denominator of η , Eq. (11), to the asymmetry has an important impact on its parameter dependence.

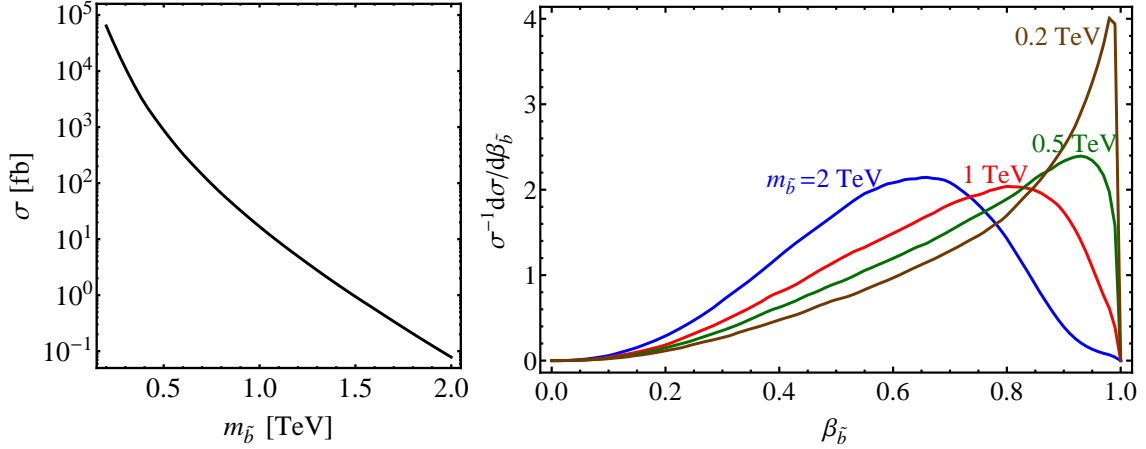


Figure 5: Total sbottom pair production cross section $\sigma(pp \rightarrow \tilde{b}\tilde{b}^*)$ at the LHC as a function of the sbottom mass (left). Normalized sbottom pair production distribution $\frac{1}{\sigma} \frac{d\sigma}{d\beta_{\tilde{b}}}$ with respect to the sbottom boost factor $\beta_{\tilde{b}}$ (right). The leading order cross sections have been calculated at $\sqrt{s_{pp}} = 14$ TeV using MadGraph [55].

3 CP asymmetries at the LHC

The production of sbottom pairs at the LHC [62]

$$p + p \rightarrow \tilde{b}_m + \tilde{b}_m^*, \quad m = 1, 2, \quad (14)$$

dominantly proceeds via gluon fusion. As a result, the leading order cross section $\sigma(pp \rightarrow \tilde{b}_m \tilde{b}_m^*)$ is independent of any other SUSY model parameters than the sbottom mass. The strong dependence can be seen in Figure 5 (left), where the cross section drops by six orders of magnitude with an increase of the squark mass from 0.2 TeV to 2 TeV. The produced sbottoms have a distinct distribution in their boost

$$\beta_{\tilde{b}} = \frac{|\mathbf{p}_{\tilde{b}}|}{E_{\tilde{b}}}, \quad (15)$$

along the direction of their momenta. In Figure 5 (right), we show the normalized boost distribution for several values for the sbottom mass. Typically, light sbottoms are highly boosted in the laboratory frame. In our scenario (Table 1), the lightest sbottom has a mass of $m_{\tilde{b}_1} = 400$ GeV and its boost distribution peaks at $\beta_{\tilde{b}_1} \approx 0.95$.

The asymmetries which are based on epsilon products \mathcal{E} , see Eqs. (7) and (8), are independent of the sbottom boost, since they are by construction Lorentz invariant. In contrast, the triple product asymmetries are not boost invariant. In Figure 6 (left), we show their boost dependence for the three possible triple product combinations $\mathcal{T} = (t\ell_1\ell_2)$, $(b\ell_1\ell_2)$, and $(t\ell_1)$. The asymmetries are calculated for $pp \rightarrow \tilde{b}_1\tilde{b}_1^*$ production at the LHC, with the subsequent decays $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$ and $\tilde{\chi}_1^- \rightarrow \ell_1\tilde{\nu}_\ell$. In the sbottom rest frame, $\beta_{\tilde{b}} = 0$, they coincide with the corresponding epsilon product asymmetries. Note that the size of the asymmetries strongly depends on the choice of momenta, and largest values are obtained for the optimal triple product $\mathcal{T} = (t\ell_1\ell_2)$, as given in Eq. (9).

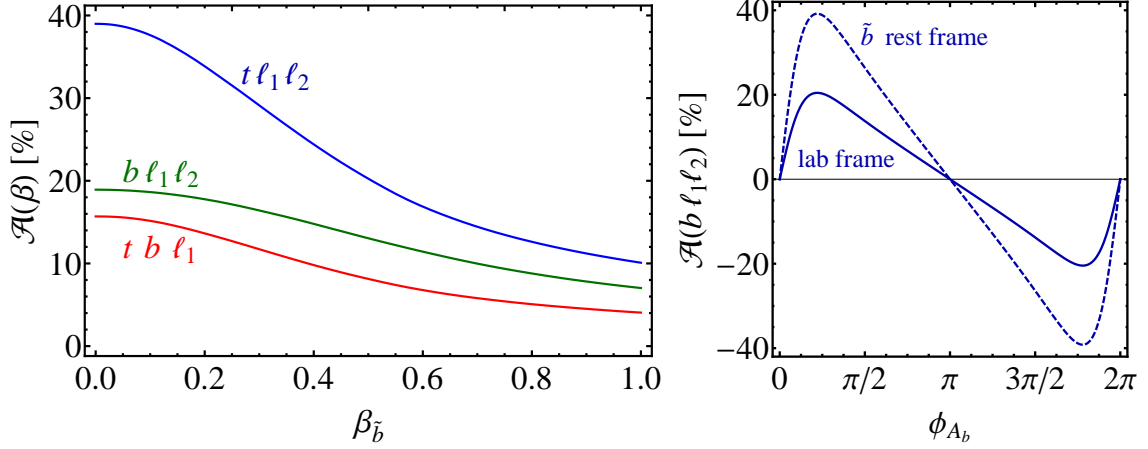


Figure 6: Triple product asymmetries $\mathcal{A}(t\ell_1\ell_2)$, $\mathcal{A}(b\ell_1\ell_2)$, and $\mathcal{A}(tb\ell_1)$, see Eq. (8), for sbottom decay, $(\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-)$, followed by $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$, and $t \rightarrow b\nu_\ell\ell_2^+$, see Figure 1), as a function of the sbottom boost $\beta_{\tilde{b}_1}$ (left). Asymmetry $\mathcal{A}(t\ell_1\ell_2)$ in the sbottom rest frame and the laboratory frame (at the LHC with $\sqrt{s} = 14$ TeV) as a function of ϕ_{A_b} (right). The SUSY parameters are given in Table 1.

3.1 Triple product asymmetries in the laboratory frame

The size of the triple product asymmetry in the laboratory (lab) frame of the LHC, is obtained by folding the boost dependent asymmetry $\mathcal{A}(\beta_{\tilde{b}})$ with the normalized sbottom boost distribution [16],

$$\mathcal{A}^{\text{lab}} = \frac{1}{\sigma} \int_0^1 \frac{d\sigma}{d\beta_{\tilde{b}}} \mathcal{A}(\beta_{\tilde{b}}) d\beta_{\tilde{b}}, \quad (16)$$

of the production cross section $\sigma = \sigma(pp \rightarrow \tilde{b}_m \tilde{b}_m^*)$. The folded asymmetry \mathcal{A}^{lab} is almost reduced by half, compared to $\mathcal{A}(\beta_{\tilde{b}} = 0)$. This can be seen in Figure 6 (right), where we show both asymmetries as a function of the CP phase ϕ_{A_b} . Thus the sbottom boost effectively reduces the asymmetries, but does not change their shape with respect to the phase dependence.

In the following, we will quantify the expected measurability of the the folded triple product asymmetry $\mathcal{A}^{\text{lab}}(t\ell_1\ell_2)$, Eq. (16), in the lab frame. Since a measurement of the asymmetries also depends on the production cross section, the sbottom boost distribution, and the size of the corresponding branching ratios for sbottoms and charginos, we will present the upper bounds on the expected luminosity,

$$\mathcal{L} = \frac{1}{\sigma} \left(\frac{1}{\mathcal{A}^2} - 1 \right). \quad (17)$$

As defined in Appendix G, \mathcal{L} is the minimal required luminosity to observe a signal at 95% CL above statistical fluctuations. In Eq. (17), the combined cross section of sbottom production and decay is denoted by σ .

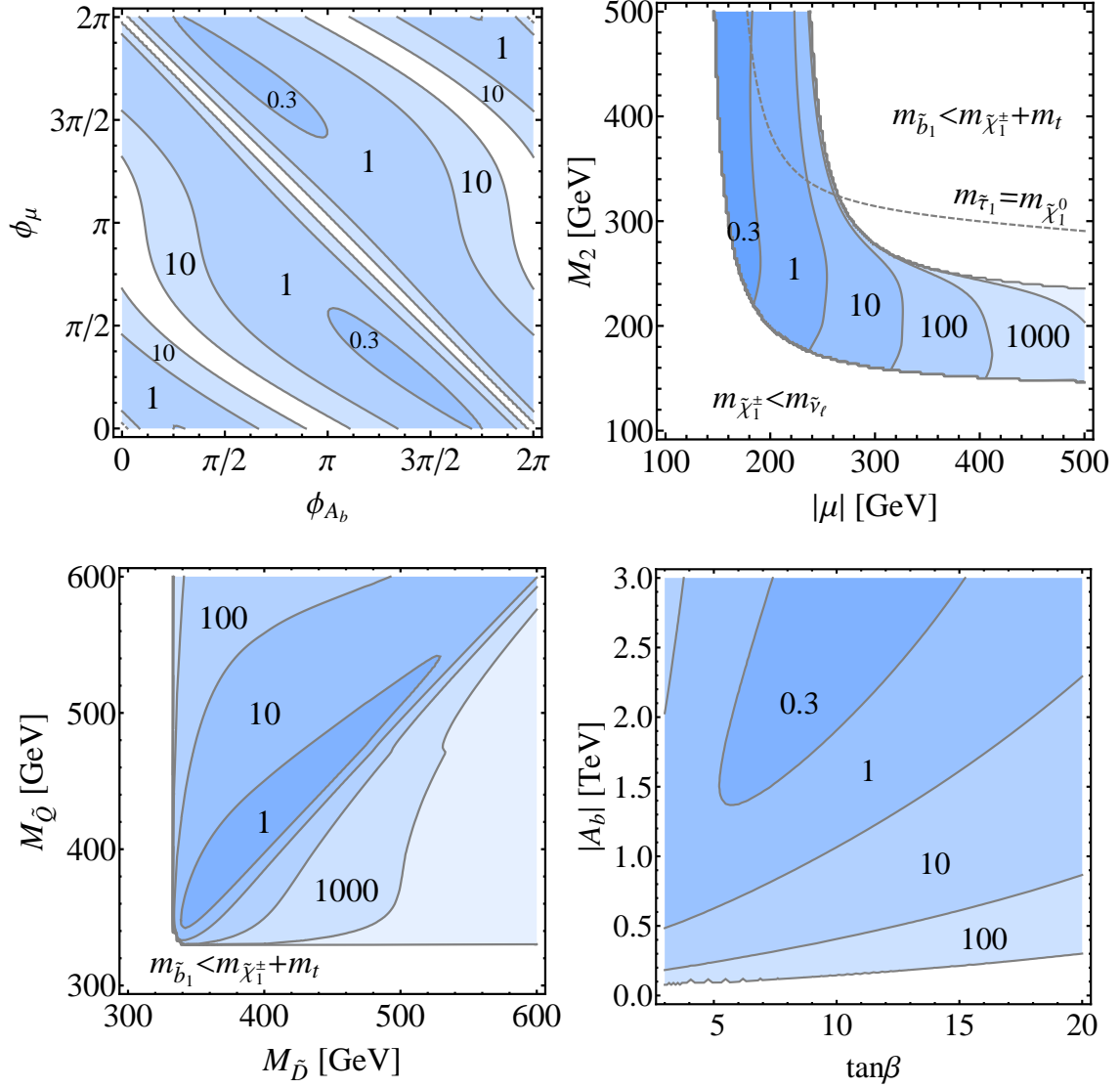


Figure 7: Contour lines of the minimal required luminosity \mathcal{L} , Eq. (17), in units of fb^{-1} to observe the triple product asymmetry $\mathcal{A}^{\text{lab}}(t\ell_1\ell_2)$, Eq. (16), at 1σ above fluctuations at the LHC, with $\sqrt{s_{pp}} = 14$ TeV, for sbottom production and decay, $\tilde{b}_1 \rightarrow t\tilde{\chi}_1^-$, followed by $\tilde{\chi}_1^- \rightarrow \ell_1^- \tilde{\nu}_\ell^*$, and $t \rightarrow b\nu_\ell \ell_2^+$. The SUSY parameters are given in Table 1.

In Figure 7, we show contour lines of the minimal required luminosity \mathcal{L} in different planes of the MSSM parameter space for the benchmark scenario as given in Table 1. The typical sizes of the sbottom and chargino branching ratios $\text{BR}(b_1 \rightarrow t\tilde{\chi}_1^\pm)$, and $\text{BR}(\tilde{\chi}_i^\pm \rightarrow e_1^\pm \tilde{\nu}_e)$, are of the order of 15% to 35%. As can be seen, a large part of the parameter space can be probed with $\mathcal{L} = 10 \text{ fb}^{-1}$.

Note however that the presented values must be regarded as absolute lower bounds on the minimal required luminosities, only. The definition of \mathcal{L} , as in Eq. (17), is purely based on the theoretical signal rate and its asymmetry, since

detector efficiency effects and contributions from CP-even backgrounds are not included. However, the minimal luminosity requirement can be used to exclude those MSSM parameter regions which cannot be probed at the LHC. Clearly, in order to give realistic values of the statistical significances and required luminosities, a detailed experimental study is necessary, which is however beyond the scope of the present work. In particular, the feasibility of the event reconstruction has to be discussed. First ideas to attempt the reconstruction of a stop decay chain have recently been reported [47].

4 Summary and conclusions

We have analyzed CP observables in the two-body decays of a light sbottom

$$\tilde{b}_1 \rightarrow t + \tilde{\chi}_1^- . \quad (18)$$

The CP-sensitive parts appear only in the top chargino spin-spin correlations, which can be probed by the subsequent decays

$$\begin{aligned} \tilde{\chi}_i^- &\rightarrow \ell_1^- + \tilde{\nu}_\ell^*, & \tilde{\nu}_\ell^* &\rightarrow \tilde{\chi}_1^0 + \bar{\nu}_\ell, & \ell &= e, \mu, \\ t &\rightarrow b + W, & W &\rightarrow \nu_\ell + \ell_2. \end{aligned} \quad (19)$$

Due to angular momentum conservation, the decay distributions of the final state momenta are correlated to each other. Asymmetries of triple products of three spatial momenta, as well as epsilon products of four space-time momenta are ideal tools to probe the CP-sensitive spin-spin correlations. The CP observables we have proposed are sensitive to the CP phases of the trilinear coupling parameter A_b , and the higgsino mass parameter μ , which might be present in the sbottom and chargino sector of the MSSM.

We have analyzed the asymmetries and event rates in a general MSSM framework. For maximal sbottom mixing, the asymmetries reach up to 40%. The class of asymmetries which are based on triple products are not Lorentz invariant, and thus are reduced by a factor of about three when evaluated in the laboratory frame at the LHC. Luminosities of at least 10 fb^{-1} are required to observe a CP signal of 1σ above statistical fluctuations at the LHC.

Clearly, the measurability of the asymmetries and thus of the CP phases can only be addressed properly in a detailed experimental analysis, which should take into account background processes, detector simulations and event reconstruction efficiencies. We want to stress the need for such a thorough analysis, to explore the potential of the LHC to probe SUSY CP violation.

Interrelations between the T-violating electric dipole moments (EDMs) and the possible SUSY CP phases underline the need to determine CP observables outside the low energy sector, in particular by measurements at the LHC. Since the proposed CP asymmetries at colliders depend in a different way on the SUSY parameters than the EDMs, they would be the ideal tools for such independent measurements. Depending on the experimental results, the EDM bounds could be either verified, or the CP-violating sectors of the underlying SUSY models have to be modified.

Acknowledgments

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Appendix

A Sbottom mixing

The masses and couplings of the sbottoms follow from their mass matrix [50]

$$\mathcal{L}_M^{\tilde{b}} = -(\tilde{b}_L^*, \tilde{b}_R^*) \begin{pmatrix} m_{\tilde{b}_L}^2 & e^{-i\phi_{\tilde{b}}} m_b |\Lambda_{\tilde{b}}| \\ e^{i\phi_{\tilde{b}}} m_b |\Lambda_{\tilde{b}}| & m_{\tilde{b}_R}^2 \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}, \quad (\text{A.1})$$

with

$$m_{\tilde{b}_L}^2 = M_{\tilde{Q}}^2 + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) m_Z^2 \cos(2\beta) + m_b^2, \quad (\text{A.2})$$

$$m_{\tilde{b}_R}^2 = M_{\tilde{D}}^2 - \frac{1}{3} m_Z^2 \sin^2 \theta_w \cos(2\beta) + m_b^2, \quad (\text{A.3})$$

with the soft SUSY-breaking parameters $M_{\tilde{Q}}$, $M_{\tilde{D}}$, the ratio $\tan \beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields, the weak mixing angle θ_w , the mass m_Z of the Z boson, and the mass m_b of the bottom quark. The CP phase of the sbottom sector is

$$\phi_{\tilde{b}} = \arg[\Lambda_{\tilde{b}}], \quad (\text{A.4})$$

$$\Lambda_{\tilde{b}} = A_b - \mu^* \tan \beta, \quad (\text{A.5})$$

with the complex trilinear scalar coupling parameter A_b , and the higgsino mass parameter μ . Note that for $|A_b| \gg |\mu| \tan \beta$ we have $\phi_{\tilde{b}} \approx \phi_{A_b}$. The sbottom mass eigenstates

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \mathcal{R}^{\tilde{b}} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}, \quad (\text{A.6})$$

are given by the diagonalization matrix [50]

$$\mathcal{R}^{\tilde{b}} = \begin{pmatrix} e^{i\phi_{\tilde{b}}} \cos \theta_{\tilde{b}} & \sin \theta_{\tilde{b}} \\ -\sin \theta_{\tilde{b}} & e^{-i\phi_{\tilde{b}}} \cos \theta_{\tilde{b}} \end{pmatrix}, \quad (\text{A.7})$$

with the sbottom mixing angle $\theta_{\tilde{b}}$

$$\cos \theta_{\tilde{b}} = \frac{-m_b |\Lambda_{\tilde{b}}|}{\sqrt{m_b^2 |\Lambda_{\tilde{b}}|^2 + (m_{\tilde{b}_1}^2 - m_{\tilde{b}_L}^2)^2}}, \quad \sin \theta_{\tilde{b}} = \frac{m_{\tilde{b}_L}^2 - m_{\tilde{b}_1}^2}{\sqrt{m_b^2 |\Lambda_{\tilde{b}}|^2 + (m_{\tilde{b}_1}^2 - m_{\tilde{b}_L}^2)^2}}. \quad (\text{A.8})$$

The mass eigenvalues are

$$m_{\tilde{b}_{1,2}}^2 = \frac{1}{2} \left[(m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2) \mp \sqrt{(m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2)^2 + 4m_b^2 |\Lambda_{\tilde{b}}|^2} \right]. \quad (\text{A.9})$$

B Chargino mixing

The complex mass matrix of the charginos is [1]

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & m_W \sqrt{2} \sin \beta \\ m_W \sqrt{2} \cos \beta & \mu \end{pmatrix}. \quad (\text{B.10})$$

with the $SU(2)$ gaugino mass parameter M_2 , and the mass of the W boson m_W . We obtain the chargino masses and the couplings by diagonalizing the chargino matrix [1],

$$U^* M_{\chi^\pm} V^\dagger = \text{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}), \quad (\text{B.11})$$

with the two independent, unitary diagonalization matrices U and V .

C Lagrangians and complex couplings

The interaction Lagrangian for sbottom decay $\tilde{b}_m \rightarrow t \tilde{\chi}_i^-$, $\tilde{b}_m^* \rightarrow \bar{t} \tilde{\chi}_i^+$, is given by [50]

$$\mathcal{L}_{t\tilde{b}\tilde{\chi}^\pm} = g \bar{t} (l_{mi}^{\tilde{b}} P_R + k_{mi}^{\tilde{b}} P_L) \tilde{\chi}_i^\pm \tilde{b}_m + \text{h.c.}, \quad (\text{C.12})$$

with $P_{L,R} = (1 \mp \gamma_5)/2$, and the weak coupling constant $g = e/\sin \theta_w$, $e > 0$. The couplings are

$$l_{mi}^{\tilde{b}} = -\mathcal{R}_{m1}^{\tilde{b}*} U_{i1} + Y_b \mathcal{R}_{m2}^{\tilde{b}*} U_{i2}, \quad k_{mi}^{\tilde{b}} = Y_t \mathcal{R}_{m1}^{\tilde{b}*} V_{i2}^*, \quad (\text{C.13})$$

with the sbottom diagonalization matrix $\mathcal{R}^{\tilde{b}}$, Eq. (A.7), the chargino diagonalization matrices U, V , Eq. (B.11), and the Yukawa couplings

$$Y_t = \frac{m_t}{\sqrt{2} m_W \sin \beta}, \quad Y_b = \frac{m_b}{\sqrt{2} m_W \cos \beta}. \quad (\text{C.14})$$

The interaction Lagrangians for chargino decay $\tilde{\chi}_i^\pm \rightarrow \ell^\pm \tilde{\nu}_\ell^{(*)}$, or $\tilde{\chi}_i^\pm \rightarrow \tilde{\ell}_L^\pm \nu_\ell$, are [1]

$$\mathcal{L}_{\ell \tilde{\nu} \tilde{\chi}_i^\pm} = -g U_{i1}^* \tilde{\chi}_i^\pm P_L \nu \tilde{\ell}_L^* - g V_{i1}^* \tilde{\chi}_i^{\pm C} P_L \ell \tilde{\nu}^* + \text{h.c.}, \quad \ell = e, \mu. \quad (\text{C.15})$$

D Kinematics and phase space

For the sbottom decay $\tilde{b}_m \rightarrow t \tilde{\chi}_i^\pm$, we choose a coordinate frame in the laboratory (lab) frame such that the momentum of the sbottom \tilde{b}_m points in the z -direction

$$p_b^\mu = (E_{\tilde{b}}, 0, 0, |\mathbf{p}_{\tilde{b}}|), \quad (\text{D.16})$$

$$p_t^\mu = (E_t, |\mathbf{p}_t| \sin \theta_t, 0, |\mathbf{p}_t| \cos \theta_t). \quad (\text{D.17})$$

The decay angle $\theta_t = \angle(\mathbf{p}_{\bar{b}}, \mathbf{p}_t)$ of the top quark is constrained by $\sin \theta_t^{\max} = |\mathbf{p}'_{\bar{b}}|/|\mathbf{p}_{\bar{b}}|$ for $|\mathbf{p}_{\bar{b}}| > |\mathbf{p}'_{\bar{b}}| = \lambda^{\frac{1}{2}}(m_{\bar{b}}^2, m_t^2, m_{\chi_i^\pm}^2)/2m_t$, with the triangle function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. In this case there are two solutions [39, 58]

$$|\mathbf{p}_t^\pm| = \frac{(m_{\bar{b}}^2 + m_t^2 - m_{\chi_i^\pm}^2)|\mathbf{p}_{\bar{b}}| \cos \theta_t \pm E_{\bar{b}} \sqrt{\lambda(m_{\bar{b}}^2, m_t^2, m_{\chi_i^\pm}^2) - 4|\mathbf{p}_{\bar{b}}|^2 m_t^2 \sin^2 \theta_t}}{2|\mathbf{p}_{\bar{b}}|^2 \sin^2 \theta_t + 2m_{\bar{b}}^2}. \quad (\text{D.18})$$

For $|\mathbf{p}_{\bar{b}}| < |\mathbf{p}'_{\bar{b}}|$ the angle θ_t is unbounded, and only the physical solution $|\mathbf{p}_t^+|$ is left.

The momenta of the subsequent decays of the chargino $\tilde{\chi}_i^\pm \rightarrow \ell_1 \tilde{\nu}_\ell^{(*)}$, Eq. (3), and those of the top quark $t \rightarrow bW$, $W \rightarrow \ell_2 \nu_\ell$, Eq. (4), can be parametrized by

$$p_b^\mu = E_b(1, \sin \theta_b \cos \phi_b, \sin \theta_b \sin \phi_b, \cos \theta_b), \quad (\text{D.19})$$

$$p_{\ell_1}^\mu = E_{\ell_1}(1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1), \quad (\text{D.20})$$

$$p_{\ell_2}^\mu = E_{\ell_2}(1, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, \cos \theta_2), \quad (\text{D.21})$$

with the energies [39]

$$E_{\ell_1} = \frac{m_{\chi_i^\pm}^2 - m_{\tilde{\nu}_\ell}^2}{2(E_{\chi_i^\pm} - |\mathbf{p}_{\chi_i^\pm}| \cos \theta_{D_1})}, \quad E_b = \frac{m_t^2 - m_W^2}{2(E_t - |\mathbf{p}_t| \cos \theta_{D_b})}, \quad (\text{D.22})$$

$$E_{\ell_2} = \frac{m_W^2}{2(E_W - |\mathbf{p}_W| \cos \theta_{D_2})}. \quad (\text{D.23})$$

The decay angles, $\theta_{D_1} = \angle(\mathbf{p}_{\chi_i^\pm}, \mathbf{p}_{\ell_1})$, $\theta_{D_b} = \angle(\mathbf{p}_t, \mathbf{p}_b)$, and $\theta_{D_2} = \angle(\mathbf{p}_W, \mathbf{p}_{\ell_2})$, are

$$\cos \theta_{D_1} = \hat{\mathbf{p}}_{\chi_i^\pm} \cdot \hat{\mathbf{p}}_{\ell_1}, \quad \cos \theta_{D_b} = \hat{\mathbf{p}}_t \cdot \hat{\mathbf{p}}_b, \quad \cos \theta_{D_2} = \hat{\mathbf{p}}_W \cdot \hat{\mathbf{p}}_{\ell_2}, \quad (\text{D.24})$$

with the unit momentum vectors $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$, and $\mathbf{p}_{\chi_i^\pm} = \mathbf{p}_{\bar{b}} - \mathbf{p}_t$, $\mathbf{p}_W = \mathbf{p}_t - \mathbf{p}_b$.

The Lorentz invariant phase-space element for the squark decay chain, see Eqs. (2)-(4), can be decomposed into two-body phase-space elements [39, 58]

$$d\text{Lips}(s_{\bar{b}}; p_{\nu_\ell}, p_{\tilde{\nu}_\ell}, p_b, p_{\ell_1}, p_{\ell_2}) = \frac{1}{(2\pi)^3} \sum_{\pm} d\text{Lips}(s_{\bar{b}}; p_t, p_{\chi_i^\pm}) \\ \times ds_{\chi_i^\pm} d\text{Lips}(s_{\chi_i^\pm}; p_{\ell_1}, p_{\tilde{\nu}_\ell}) ds_t d\text{Lips}(s_t; p_b, p_W) ds_W d\text{Lips}(s_W; p_{\ell_2}, p_{\nu_\ell}), \quad (\text{D.25})$$

where we have to sum the two solutions $|\mathbf{p}_t^\pm|$ of the top quark momentum, see Eq. (D.18), if the decay angle θ_t is constrained. The different factors are

$$d\text{Lips}(s_{\bar{b}}; p_t, p_{\chi_i^\pm}) = \frac{1}{8\pi} \frac{|\mathbf{p}_t|^2}{|E_t |\mathbf{p}_{\bar{b}}| \cos \theta_t - E_{\bar{b}} |\mathbf{p}_t|} \sin \theta_t d\theta_t, \quad (\text{D.26})$$

$$d\text{Lips}(s_{\chi_i^\pm}; p_{\ell_1}, p_{\tilde{\nu}_\ell}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\ell_1}|^2}{m_{\chi_i^\pm}^2 - m_{\tilde{\nu}_\ell}^2} d\Omega_1, \quad (\text{D.27})$$

$$d\text{Lips}(s_t; p_b, p_W) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_b|^2}{m_t^2 - m_W^2} d\Omega_b, \quad (\text{D.28})$$

$$d\text{Lips}(s_W; p_{\ell_2}, p_{\nu_\ell}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\ell_2}|^2}{m_W^2} d\Omega_3, \quad (\text{D.29})$$

with $s_j = p_j^2$ and $d\Omega_j = \sin \theta_j d\theta_j d\phi_j$. We use the narrow width approximation

$$\int |\Delta(j)|^2 ds_j = \frac{\pi}{m_j \Gamma_j}, \quad (\text{D.30})$$

for the propagators

$$\Delta(j) = \frac{i}{s_j - m_j^2 + im_j \Gamma_j}, \quad (\text{D.31})$$

which is justified for $\Gamma_j/m_j \ll 1$, which holds in our case for particle widths $\Gamma_j \lesssim \mathcal{O}(1 \text{ GeV})$, and masses $m_j \approx \mathcal{O}(100 \text{ GeV})$. Note, however, that the naive $\mathcal{O}(\Gamma/m)$ -expectation of the error can easily receive large off-shell corrections of an order of magnitude and more, in particular at threshold, or due to interferences with other resonant or non-resonant processes. For a recent discussion of these issues, see, for example, Ref. [59].

E Density matrix formalism

The amplitude squared for the entire sbottom decay chain, Eqs. (2)-(4), has been calculated in Ref. [50], by using the spin formalism of Kawasaki, Shirafuji and Tsai [60]. We calculate the amplitude squared in the spin-density matrix formalism [52, 61], which allows a separation of the amplitude squared into contributions from spin correlations, spin-spin correlations and from the unpolarized part. In that way, the CP-sensitive parts of the amplitude squared can be easily separated and identified, allowing to find the epsilon product, that yields the largest CP asymmetries.

In the spin-density matrix formalism of Ref. [52], the amplitude squared of the sbottom decay chain, Eqs. (2)-(4), can be written as

$$|T|^2 = |\Delta(t)|^2 |\Delta(\tilde{\chi}_i^-)|^2 |\Delta(W)|^2 \times \sum_{\lambda_i, \lambda'_i, \lambda_t, \lambda'_t, \lambda_k, \lambda'_k} \rho_D(\tilde{b})_{\lambda_i \lambda'_i}^{\lambda_t \lambda'_t} \rho_{D_1}(\tilde{\chi}_i^\pm)^{\lambda'_i \lambda_i} \rho_{D_2}(t)_{\lambda'_t \lambda_t}^{\lambda'_k \lambda_k} \rho_{D_3}(W)_{\lambda_k \lambda'_k}. \quad (\text{E.32})$$

The amplitude squared is composed of the propagators $\Delta(j)$, Eq. (D.31), of particle $j = t, \tilde{\chi}_i^\pm$, or W , and the un-normalized spin density matrices $\rho_D(\tilde{b})$, $\rho_{D_1}(\tilde{\chi}_i^\pm)$, $\rho_{D_2}(t)$, and $\rho_{D_3}(W)$, with the helicity indices λ_i, λ'_i of the chargino, the helicity indices λ_t, λ'_t of the top quark, and those of the W boson, λ_k, λ'_k .

The density matrices can be expanded in terms of the Pauli matrices

$$\begin{aligned} \rho_D(\tilde{b})_{\lambda_i \lambda'_i}^{\lambda_t \lambda'_t} &= \delta^{\lambda_t \lambda'_t} \delta_{\lambda_i \lambda'_i} D + \delta_{\lambda_i \lambda'_i} (\sigma^a)^{\lambda_t \lambda'_t} \Sigma_D^a + \delta^{\lambda_t \lambda'_t} (\sigma^b)_{\lambda_i \lambda'_i} \Sigma_D^b \\ &\quad + (\sigma^a)^{\lambda_t \lambda'_t} (\sigma^b)_{\lambda_i \lambda'_i} \Sigma_D^{ab}, \end{aligned} \quad (\text{E.33})$$

$$\rho_{D_1}(\tilde{\chi}_i^\pm)^{\lambda'_i \lambda_i} = \delta^{\lambda'_i \lambda_i} D_1 + (\sigma^b)^{\lambda'_i \lambda_i} \Sigma_{D_1}^b, \quad (\text{E.34})$$

$$\rho_{D_2}(t)_{\lambda'_t \lambda_t}^{\lambda'_k \lambda_k} = [\delta_{\lambda'_t \lambda_t} D_2^{\mu\nu} + (\sigma^a)_{\lambda'_t \lambda_t} \Sigma_{D_2}^{a\mu\nu}] \varepsilon_{\mu}^{\lambda'_k *} \varepsilon_{\nu}^{\lambda'_k}, \quad (\text{E.35})$$

$$\rho_{D_3}(W)_{\lambda'_k \lambda_k}^{\lambda'_k \lambda_k} = D_3^{\rho\sigma} \varepsilon_{\rho}^{\lambda_k} \varepsilon_{\sigma}^{\lambda'_k *}, \quad (\text{E.36})$$

with an implicit sum over $a, b = 1, 2, 3$.

The polarization vectors $\varepsilon_\mu^{\lambda_k}$ of the W boson fulfil the completeness relation

$$\sum_{\lambda_k} \varepsilon_\mu^{\lambda_k*} \varepsilon_\nu^{\lambda_k} = -g_{\mu\nu} + \frac{p_{W,\mu} p_{W,\nu}}{m_W^2}, \quad (\text{E.37})$$

with $p_W^\mu \varepsilon_\mu^{\lambda_k} = 0$. Similarly the spin four-vectors s_t^a , $a = 1, 2, 3$, for the top quark t , and $s_{\chi_i^\pm}^b$, $b = 1, 2, 3$, for the chargino $\tilde{\chi}_i^\pm$, also fulfil completeness relations

$$\sum_a s_t^{a,\mu} s_t^{a,\nu} = -g^{\mu\nu} + \frac{p_t^\mu p_t^\nu}{m_t^2}, \quad \sum_b s_{\chi_i^\pm}^{b,\mu} s_{\chi_i^\pm}^{b,\nu} = -g^{\mu\nu} + \frac{p_{\chi_i^\pm}^\mu p_{\chi_i^\pm}^\nu}{m_{\chi_i^\pm}^2}, \quad (\text{E.38})$$

and they form an orthonormal set

$$s_t^a \cdot s_t^b = -\delta^{ab}, \quad s_t^a \cdot \hat{p}_t = 0, \quad s_{\chi_i^\pm}^a \cdot s_{\chi_i^\pm}^b = -\delta^{ab}, \quad s_{\chi_i^\pm}^a \cdot \hat{p}_{\chi_i^\pm} = 0, \quad (\text{E.39})$$

with the notation $\hat{p}^\mu = p^\mu/m$.

The expansion coefficients of the matrices, Eqs. (E.33)-(E.35), are

$$D = \frac{g^2}{2} \left(|\tilde{l}_{mi}^b|^2 + |\tilde{k}_{mi}^b|^2 \right) (p_t \cdot p_{\chi_i^\pm}) - g^2 \text{Re}\{\tilde{l}_{mi}^b (\tilde{k}_{mi}^b)^*\} m_t m_{\chi_i^\pm}, \quad (\text{E.40})$$

$$\Sigma_D^a = \frac{(-)}{(+)} \frac{g^2}{2} \left(|\tilde{l}_{mi}^b|^2 - |\tilde{k}_{mi}^b|^2 \right) m_t (p_{\chi_i^\pm} \cdot s_t^a), \quad (\text{E.41})$$

$$\Sigma_D^b = \frac{(-)}{(+)} \frac{g^2}{2} \left(|\tilde{l}_{mi}^b|^2 - |\tilde{k}_{mi}^b|^2 \right) m_{\chi_i^\pm} (p_t \cdot s_{\chi_i^\pm}^b), \quad (\text{E.42})$$

$$\begin{aligned} \Sigma_D^{ab} = & \frac{g^2}{2} \left(|\tilde{l}_{mi}^b|^2 + |\tilde{k}_{mi}^b|^2 \right) (s_t^a \cdot s_{\chi_i^\pm}^b) m_t m_{\chi_i^\pm} \\ & + g^2 \text{Re}\{\tilde{l}_{mi}^b (\tilde{k}_{mi}^b)^*\} \left[(s_t^a \cdot p_{\chi_i^\pm}) (s_{\chi_i^\pm}^b \cdot p_t) - (s_t^a \cdot s_{\chi_i^\pm}^b) (p_{\chi_i^\pm} \cdot p_t) \right] \\ & - g^2 \text{Im}\{\tilde{l}_{mi}^b (\tilde{k}_{mi}^b)^*\} [s_t^a, p_t, s_{\chi_i^\pm}^b, p_{\chi_i^\pm}], \end{aligned} \quad (\text{E.43})$$

$$D_1 = \frac{g^2}{2} |V_{i1}|^2 (m_{\chi_i^\pm}^2 - m_{\tilde{\nu}_\ell}^2), \quad (\text{E.44})$$

$$\Sigma_{D_1}^b = \frac{(+)}{(-)} g^2 |V_{i1}|^2 m_{\chi_i^\pm} (s_{\chi_i^\pm}^b \cdot p_{\ell_1}), \quad (\text{E.45})$$

$$D_2^{\mu\nu} = \frac{g^2}{2} [p_b^\mu p_t^\nu + p_b^\nu p_t^\mu - (p_b \cdot p_t) g^{\mu\nu}] \frac{(+)}{(-)} \frac{g^2}{2} i \varepsilon^{\mu\alpha\nu\beta} p_{t,\alpha} p_{b,\beta}, \quad (\text{E.46})$$

$$\Sigma_{D_2}^{a\mu\nu} = \frac{(-)}{(+)} \frac{g^2}{2} m_t \left\{ [p_b^\mu s_t^{a,\nu} + p_b^\nu s_t^{a,\mu} - (p_b \cdot s_t^a) g^{\mu\nu}] \frac{(+)}{(-)} i \varepsilon^{\mu\alpha\nu\beta} s_{t,\alpha} p_{b,\beta} \right\}, \quad (\text{E.47})$$

$$D_3^{\rho\sigma} = g^2 [p_{\ell_2}^\rho p_{\nu_\ell}^\sigma + p_{\ell_2}^\sigma p_{\nu_\ell}^\rho - (p_{\ell_2} \cdot p_{\nu_\ell}) g^{\rho\sigma}] \frac{(+)}{(-)} g^2 i \varepsilon^{\rho\alpha\sigma\beta} p_{\ell_2,\alpha} p_{\nu_\ell,\beta}, \quad (\text{E.48})$$

with the couplings as defined in Appendix C, and the short hand notation

$$[p_1, p_2, p_3, p_4] \equiv \varepsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha p_4^\beta; \quad \varepsilon_{0123} = 1. \quad (\text{E.49})$$

The coefficients for sbottom decay, Eqs. (E.40)-(E.43), are obtained from those of stop decay given in Ref. [16], by the replacements of the couplings $a_{mi}^{\tilde{t}} \rightarrow l_{mi}^{\tilde{b}}$, and $b_{mi}^{\tilde{t}} \rightarrow k_{mi}^{\tilde{b}}$. The signs in parentheses hold for the charge conjugated processes, that is $\tilde{b}_m^* \rightarrow \tilde{t}\tilde{\chi}_i^+$ in Eqs. (E.41) and (E.42), $\tilde{\chi}_i^+ \rightarrow \ell_1^+\tilde{\nu}_\ell^*$ in Eq. (E.45), $\tilde{t} \rightarrow \tilde{b}W^-$ in Eqs. (E.46) and (E.47), and finally $W^- \rightarrow \bar{\nu}_\ell \ell_2^-$ in Eq. (E.48).

Inserting the density matrices, Eqs. (E.33)-(E.36) into Eq. (E.32), we obtain

$$|T|^2 = 4 |\Delta(t)|^2 |\Delta(\tilde{\chi}_i^\pm)|^2 |\Delta(W)|^2 \times$$

$$[D \ D_1 \ D_2^{\rho\sigma} + D_1 \ \Sigma_D^a \ \Sigma_{D_2}^{a\rho\sigma} + \Sigma_D^b \ \Sigma_{D_1}^b \ D_2^{\rho\sigma} + \Sigma_D^{ab} \ \Sigma_{D_1}^b \ \Sigma_{D_2}^{a\rho\sigma}] D_{3\rho\sigma}, \quad (\text{E.50})$$

with an implicit sum over a, b . The amplitude squared $|T|^2$ is now composed into an unpolarized part (first summand), into the spin correlations of the top (second summand), those of the chargino (third summand), and into the spin-spin correlations of top and chargino (fourth summand), in Eq. (E.50).

With the completeness relation for the W polarization vectors, Eq. (E.37), we find

$$D_2^{\rho\sigma} D_{3\rho\sigma} = 2g^4(p_t \cdot p_{\ell_2})(p_b \cdot p_{\nu_\ell}), \quad (\text{E.51})$$

$$\Sigma_{D_2}^{a\rho\sigma} D_{3\rho\sigma} = \begin{matrix} - \\ (+) \end{matrix} 2m_t g^4(s_t^a \cdot p_{\ell_2})(p_b \cdot p_{\nu_\ell}), \quad (\text{E.52})$$

and the sign in parenthesis for the charge conjugated decay, $\tilde{t} \rightarrow \tilde{b}W^-$, $W^- \rightarrow \bar{\nu}_\ell \ell_2^-$. By also using the completeness relations for the top and chargino spin vectors, Eq. (E.38), the products in Eq. (E.50) can be written as

$$\begin{aligned} \Sigma_D^a \Sigma_{D_2}^{a\rho\sigma} D_{3\rho\sigma} &= g^6 \left(|l_{mi}^{\tilde{b}}|^2 - |k_{mi}^{\tilde{b}}|^2 \right) (p_b \cdot p_{\nu_\ell}) \\ &\times \left[(p_{\chi_i^\pm} \cdot p_t)(p_{\ell_2} \cdot p_t) - m_t^2(p_{\chi_i^\pm} \cdot p_{\ell_2}) \right], \end{aligned} \quad (\text{E.53})$$

$$\begin{aligned} \Sigma_D^b \Sigma_{D_1}^b &= \frac{g^4}{2} \left(|l_{mi}^{\tilde{b}}|^2 - |k_{mi}^{\tilde{b}}|^2 \right) |V_{i1}|^2 \\ &\times \left[m_{\chi_i^\pm}^2(p_t \cdot p_{\ell_1}) - (p_{\chi_i^\pm} \cdot p_t)(p_{\ell_1} \cdot p_{\chi_i^\pm}) \right], \end{aligned} \quad (\text{E.54})$$

$$\begin{aligned} \Sigma_D^{ab} \Sigma_{D_1}^b \Sigma_{D_2}^{a\rho\sigma} D_{3\rho\sigma} &= g^8 \left(|l_{mi}^{\tilde{b}}|^2 + |k_{mi}^{\tilde{b}}|^2 \right) |V_{i1}|^2 \left[(p_{\chi_i^\pm} \cdot p_{\ell_1})(p_{\chi_i^\pm} \cdot p_{\ell_2})m_t^2 \right. \\ &+ (p_t \cdot p_{\ell_1})(p_t \cdot p_{\ell_2})m_{\chi_i^\pm}^2 - (p_{\ell_1} \cdot p_{\ell_2})m_{\chi_i^\pm}^2 m_t^2 \\ &\left. - (p_{\chi_i^\pm} \cdot p_t)(p_t \cdot p_{\ell_2})(p_{\chi_i^\pm} \cdot p_{\ell_1}) \right] (p_b \cdot p_{\nu_\ell}) \\ &+ 2g^8 \text{Re}\{l_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\} |V_{i1}|^2 m_{\chi_i^\pm} m_t (p_b \cdot p_{\nu_\ell}) \\ &\times \left[(p_{\chi_i^\pm} \cdot p_t)(p_{\ell_1} \cdot p_{\ell_2}) - (p_{\chi_i^\pm} \cdot p_{\ell_2})(p_t \cdot p_{\ell_1}) \right] \\ &- 2g^8 \text{Im}\{l_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\} |V_{i1}|^2 m_{\chi_i^\pm} m_t (p_b \cdot p_{\nu_\ell}) [p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}], \end{aligned} \quad (\text{E.55})$$

with the short hand notation Eq. (E.49). There is no sign change in the terms Eqs. (E.53)-(E.55) for the charge conjugated process $\tilde{b}_m^* \rightarrow \tilde{t}\tilde{\chi}_i^+$, with the subsequent decays $\tilde{\chi}_i^+ \rightarrow \ell_1^+\tilde{\nu}_\ell^*$, and $\tilde{t} \rightarrow \tilde{b}W^-$, $W^- \rightarrow \bar{\nu}_\ell \ell_2^-$.

F Sbottom decay widths and asymmetry

The partial decay width for the sbottom decay $\tilde{b}_m \rightarrow t\tilde{\chi}_i^\pm$ is [26]

$$\Gamma(\tilde{b}_m \rightarrow t\tilde{\chi}_i^\pm) = \frac{\sqrt{\lambda(m_{\tilde{b}}^2, m_t^2, m_{\tilde{\chi}_i^\pm}^2)}}{4\pi m_{\tilde{b}}^3} D, \quad (\text{F.56})$$

with the decay function D given in Eqs. (E.40). For the decay $\tilde{b}_m \rightarrow b\tilde{\chi}_j^0$ we have [26]

$$\Gamma(\tilde{b}_m \rightarrow b\tilde{\chi}_j^0) = \frac{(m_{\tilde{b}}^2 - m_{\tilde{\chi}_j^0}^2)^2}{16\pi m_{\tilde{b}}^3} g^2 \left(|a_{mj}^{\tilde{b}}|^2 + |b_{mj}^{\tilde{b}}|^2 \right), \quad (\text{F.57})$$

with the approximation $m_b = 0$. The sbottom-bottom-neutralino couplings are [26]

$$a_{mj}^{\tilde{b}} = \mathcal{R}_{m1}^{\tilde{b}*} f_{bj}^L + \mathcal{R}_{m2}^{\tilde{b}*} h_{bj}^R, \quad b_{mj}^{\tilde{b}} = \mathcal{R}_{m1}^{\tilde{b}*} h_{bj}^L + \mathcal{R}_{m2}^{\tilde{b}*} f_{bj}^R, \quad (\text{F.58})$$

$$f_{bj}^L = -\sqrt{2} \left[\frac{1}{\cos \theta_w} \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w \right) N_{j2} - \frac{1}{3} \sin \theta_w N_{j1} \right], \quad (\text{F.59})$$

$$f_{bj}^R = \frac{\sqrt{2}}{3} \sin \theta_w (\tan \theta_w N_{j2}^* - N_{j1}^*), \quad (\text{F.60})$$

$$h_{bj}^L = (h_{bj}^R)^* = -Y_b (\cos \beta N_{j3}^* + \sin \beta N_{j4}^*), \quad (\text{F.61})$$

with the sbottom diagonalization matrix $\mathcal{R}^{\tilde{b}}$, Eq. (A.7), and N the diagonalization matrix for the neutralino matrix in the photino, zino, higgsino basis, see Ref. [16]

The sbottom decay width for the complete decay chain, Eqs. (2)-(4), is given by

$$\Gamma(\tilde{b} \rightarrow \nu_\ell \tilde{\nu}_\ell \tilde{\chi}_i^\pm b \ell_1 \ell_2) = \frac{1}{2m_{\tilde{b}}} \int |T|^2 d\text{Lips}(s_{\tilde{b}}; p_{\nu_\ell}, p_{\tilde{\nu}_\ell}, p_{\tilde{\chi}_i^\pm}, p_b, p_{\ell_1}, p_{\ell_2}), \quad (\text{F.62})$$

with the phase-space element $d\text{Lips}$ given in Eq. (D.25).

We obtain an explicit expression for the asymmetry, if we insert the amplitude squared $|T|^2$, Eq. (E.50), into Eq. (8),

$$\mathcal{A} = \frac{\int \text{Sign}(\mathcal{E}) \Sigma_D^{ab} \Sigma_{D_1}^b \Sigma_{D_2}^{a\rho\sigma} D_{3\rho\sigma} d\text{Lips}}{\int D D_1 D_2^{\rho\sigma} D_{3\rho\sigma} d\text{Lips}}, \quad (\text{F.63})$$

where we have already used the narrow width approximation of the propagators, see Eq. (D.30). In the numerator, only the spin-spin terms of the amplitude squared remain, since only they contain the epsilon product \mathcal{E} , see Eq. (7). The other terms vanish due to the phase-space integration over the sign of the epsilon product, $\text{Sign}(\mathcal{E})$. In the denominator, all spin and spin-spin correlation terms vanish, and only the spin-independent parts contribute. Inserting now the explicit expressions of the terms Eqs. (E.40)-(E.55) into the formula for the asymmetry, Eq. (F.63), we find

$$\mathcal{A} = \eta \frac{\int \text{Sign}(\mathcal{E}) (p_b \cdot p_{\nu_\ell}) [p_{\tilde{b}}, p_t, p_{\ell_1}, p_{\ell_2}] d\text{Lips}}{(p_{\tilde{\chi}_i^\pm} \cdot p_{\ell_1}) \int (p_t \cdot p_{\ell_2}) (p_b \cdot p_{\nu_\ell}) d\text{Lips}}, \quad (\text{F.64})$$

with the coupling function

$$\eta = \frac{\text{Im}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}}{\frac{1}{2}\left(|\tilde{l}_{mi}^{\tilde{b}}|^2 + |k_{mi}^{\tilde{b}}|^2\right) \frac{m_b^2 - m_{\chi_i^\pm}^2 - m_t^2}{2m_{\chi_i^\pm} m_t} - \text{Re}\{\tilde{l}_{mi}^{\tilde{b}}(k_{mi}^{\tilde{b}})^*\}}. \quad (\text{F.65})$$

G Theoretical statistical significance

Assuming that the fluctuations of the signal rate are binomially distributed with the selection probability $p = 1/2(\mathcal{A} + 1)$, the significance of an asymmetry is [16]

$$\mathcal{S} = \frac{|\mathcal{A}|}{\sqrt{1 - \mathcal{A}^2}} \sqrt{\sigma \mathcal{L}}, \quad (\text{G.66})$$

with the integrated LHC luminosity \mathcal{L} , and the cross section σ of sbottom production and decay as defined below. The statistical significance \mathcal{S} is equal to the number of standard deviations that the asymmetry can be statistically determined to be non-zero. For example a value of $\mathcal{S} = 1$ implies a measurement at the 68% confidence level. The minimal required luminosity is then

$$\mathcal{L} = \frac{1}{\sigma} \left(\frac{1}{\mathcal{A}^2} - 1 \right). \quad (\text{G.67})$$

The cross section for sbottom production and decay is

$$\begin{aligned} \sigma = & F_N \times \sigma(pp \rightarrow \tilde{b}_m \tilde{b}_m^*) \times \text{Br}(\tilde{b}_m \rightarrow t \tilde{\chi}_i^-) \times \text{Br}(t \rightarrow bW) \times \text{Br}(W \rightarrow \nu_e e) \\ & \times \text{Br}(\tilde{\chi}_i^- \rightarrow e^+ \tilde{\nu}_e) \times \text{Br}(\tilde{\nu}_e \rightarrow \tilde{\chi}_1^0 \nu_e), \end{aligned} \quad (\text{G.68})$$

for $m = 1, 2$, and $i = 1, 2$. For m, i fixed, the combinatorial factor F_N takes into account the possible W and chargino $\tilde{\chi}_i^-$ decays into leptons with different flavors. We assume that the branching ratios do not depend on the flavor, i.e., $\text{Br}(W \rightarrow \nu_e e) = \text{Br}(W \rightarrow \nu_\mu \mu)$, and $\text{Br}(\tilde{\chi}_i^- \rightarrow e^+ \tilde{\nu}_e) = \text{Br}(\tilde{\chi}_i^- \rightarrow \mu^+ \tilde{\nu}_\mu)$. The factor is thus $F_N = 4$, if we sum the lepton flavors e, μ . We further assume $\text{Br}(\tilde{\nu}_e \rightarrow \tilde{\chi}_1^0 \nu_e) = 1$, which applies to our SUSY scenarios considered.

For the calculation of the sbottom decay widths and branching ratios, we use the formulas as given in Eqs. (F.56), (F.57). For the calculation of the chargino decay widths and branching ratios, we consider the two-body decays [39]

$$\tilde{\chi}_1^\pm \rightarrow \ell \tilde{\nu}_\ell, \tau \tilde{\nu}_\tau, \tilde{\ell}_L \nu_\ell, \tilde{\tau}_{1,2} \nu_\tau, W \tilde{\chi}_1^0, \quad \ell = e, \mu. \quad (\text{G.69})$$

We neglect three-body decays, which are suppressed by phase space.

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